Preprint typeset in JHEP style. - PAPER VERSION

hep-th/01??
RUNHETC-2001-5
SCIPP-01/2
UTTG-02-01

M-theory observables for cosmological space-times

T. Banks*
Department of Physics
University of California, Santa Cruz, CA 95064
E-mail: banks@scipp.ucsc.edu

W. Fischler
Department of Physics
University of Texas, Austin, TX 78712
E-mail: fischler@physics.utexas.edu

ABSTRACT: We discuss the construction of the analog of an S-matrix for space-times that begin with a Big-Bang and asymptote to an FRW universe with nonnegative cosmological constant. When the cosmological constant is positive there are many such S-matrices, related mathematically by gauge transformations and physically by an analog of the principle of black hole complementarity. In the limit of vanishing Λ these become (approximate) Poincare transforms of each other. Considerations of the initial state require a quantum treatment of space-time, and some preliminary steps towards constructing such a theory are proposed. In this context we propose a model for the earliest semiclassical state of the universe, which suggests a solution for the horizon problem different from that provided by inflation.

KEYWORDS: M-Theory, Cosmology.

*On leave from Rutgers U.

Contents

1. Introduction

In perturbative string theory, the only gauge invariant observable is the S-matrix. In modern times [4], this has been taken as evidence that the theory obeys the holographic principle [5]. In asymptotically flat spacetimes we can only probe it by sources localized on null infinity. In asymptotically Anti-DeSitter spacetimes, probes are similarly restricted to the boundary. The current technical machinery for understanding holography relies on a holographic screen at infinity. This is of course problematic when one tries to apply it to cosmology, particularly if we want to discuss closed universes. We have no doubt that the ultimate resolution of this puzzle will involve the recognition of a new gauge principle under which one can change the holographic screen without changing the physics. Indeed, the oldest holographic descriptions of M-theory vacua, Thorne's string bit models [6] and Matrix Theory [7] use a null hyperplane at infinity as the screen, via a choice of light cone gauge. It is clear that if the theory is Lorentz invariant we must be able to change the screen by a gauge transformation (*i.e.* pick a

different light cone gauge). Much more definite evidence for the gauge nature of the choice of holographic screen comes from Bousso's work on entropy bounds in general spacetimes [8]. There it was shown that the entropy (and thus the quantum states) of a general spacetime satisfying Einstein's equations and the dominant energy condition, could be assigned to a collection of holographic screens whose area counted the entropy. However, there is an enormous freedom in how the screens are chosen.

It seems clear that a more local description of the theory will require us to obtain a deeper understanding of this new gauge invariance[1]. One hint that was suggested in the talk by one of the authors (TB) at Strings at the Millenium [9] is a relation between this symmetry and local supersymmetry, via the twistor transform. We hope to return to a detailed discussion of this connection in a future publication [10]. For now suffice it to say that the choice of a (pure) spinor at a point P of spacetime or some brane embedded in it is equivalent to the choice of a null direction and an infinitesimal hyperplane transverse to it. This hyperplane should be thought of as the bit of the holographic screen on which the data at P is projected, and a gauge transformation which allows one to change the spinor is the required holographic gauge invariance. It is related to local SUSY in spacetime, and κ symmetry on branes. The gauge variant nature of local physics is a generalization of the notorious *problem of time* in canonically quantized general relativity. It implies that many of the usual notions of local physics only make sense approximately, at low energies, and in regions where the spacetime curvature is small. What then are the mathematically well defined, gauge invariant observables of cosmological spacetimes? This is the question we will attempt to answer, for the case of asymptotically expanding universes, in the current paper.

Morally speaking, our proposal can be defined in the classical approximation in terms of solutions of the field equations with boundary conditions on the Big Bang singularity and on a null surface in the future (null infinity if $\Lambda = 0$ and the future cosmological horizon(s) if the spacetime is Asymptotically DeSitter (AsDS)). There are two apparent problems with this description of the observables. The Big Bang is singular and the boundary value problem is not well defined there. In AsDS spaces, there are *many* cosmological horizons. The first of these problems will be solved when we have a complete quantum description of geometry. Some first tentative steps in this direction will be taken in this paper. The problem of multiple horizons in AsDS spaces will be seen to be related to the principle of Black Hole Complementarity [18]. This in turn is related to the *problem of time*: different observers in quantum spacetimes, have evolution operators that do not commute.

[1]L. Susskind has long advocated that a local description of M-theory would involve the introduction of many gauge degrees of freedom.

It has been argued [3] that AsDS spaces are described by quantum systems with a finite dimensional Hilbert space, with dimension equal to the exponential of the Hawking-DeSitter entropy. Bousso [11] has recently proven that no experiment in a spacetime with positive cosmological constant can probe more information than this. We will argue that although there is no truly gauge independent physical S-matrix in AsDS spaces, there are gauge dependent objects which become invariant in the limit in which the cosmological constant vanishes. Close to this limit these objects represent physics as seen by low energy observers who have passed outside each other's horizon. The relation between them is similar to that between infalling and asymptotic observers for a very large black hole. We argue that the finite number of AsDS states is compatible with the indefinite growth of the number of horizon volumes in AsDS space because of a principle analogous to black hole complementarity. According to this principle, the Hilbert space describing the interior of a black hole is a tensor factor of the Hilbert space of the observer at infinity[2]. The radical differences in the physics described by these observers is attributed to noncommutativity of the observables that they measure. Similarly, each observer in AsDS space sees everything in the space that is not bound to her, being absorbed in her cosmological horizon. Since the physics in these different horizon volumes is causally disconnected, there is nothing to stop us from using different bases of the same Hilbert space to describe all of them[3]. Thus, in an AsDS space, we will define an infinite number of different S-matrices, related by unitary transformations. As $\Lambda \rightarrow 0$ and the cosmological horizon recedes to infinity, these transformations approximately approach Poincare transformations relating observers at different points and in different Lorentz frames of an asymptotically flat space (though a real cosmology will never be exactly asymptotically flat and Poincare invariant).

The rest of this paper is organized as follows. In the next section we formulate our prescription for the S-matrix in supergravity (SUGRA) language as well as for toroidal cosmologies of weakly coupled superstrings. We emphasize that all of these descriptions are inadequate because any cosmological solution contains a regime (which we call the Big Bang) where all weakly coupled or low energy descriptions fail. We give a preliminary semiclassical discussion of the physics of the Big Bang regime and show that it is dominated by matter satisfying the equation of state $p = \rho$, which is the equation of the homogeneous modes of moduli. This is the stiffest equation of state for which the speed of sound is less than or equal to that of light. It is also the stiffest equation of state [12] for which the holographic principle can be satisfied at arbitrarily early times. We argue that moduli are not a satisfactory description of the Big Bang

[2]All of this language is appropriate to time scales short compared to the black hole lifetime.
[3]This sentence is shorthand for the extensive discussion and explanation to be found in [18].

regime because they cannot saturate the entropy. A model which does saturate the entropy is one which postulates a $1+1$ dimensional conformal field theory (whose central charge depends on the spacetime dimension) in each Planck scale volume of spacetime. The $L_0 + \bar{L_0}$ eigenvalue of the CFT is related to the initial energy density in Planck units. The CFT's in different Planck volumes are constrained to have the same eigenvalue of $L_0 + \bar{L_0}$. We also note that this model for the earliest state of the universe suggests a solution for the horizon problem different from that provided by inflation. This is implicit in the statement that homogeneous energy and entropy densities satisfying $p = \rho$ can saturate the maximal entropy allowed by the holographic principle. This means that a generic state of the universe near the Big Bang will have a homogeneous pressure, energy and entropy density in comoving coordinates, although its other characteristics need not be homogeneous (*i.e.* the state of the CFT in each Planck volume will have the same $L_0 + \bar{L_0}$ eigenvalue, but will not be the same state).

The third section contains speculative material relevant for the construction of a true quantum theory of gravity. We argue that nets of finite dimensional Hilbert spaces (equivalently, finite dimensional C^* algebras) can encode both the causal and metric properties of a spacetime satisfying the dominant energy condition and give a generalization of the notion of geometry to high curvature regimes that is based on nothing but the fundamentals of quantum mechanics. A Lorentzian manifold is determined by its causal structure, its conformal factor and its dimension. We suggest that near the Big Bang the dimension of spacetime is determined by the maximum rate at which information can be accumulated by an observer (this occurs for the $p = \rho$ equation of state) and propose a way to match this to the behavior of nets of Hilbert spaces. The material in this section bears much resemblance to [4], previous attempts to construct discrete theories of quantum gravity [2]. In the fourth section we discuss the generalization of these considerations to AsDS spaces and explain why the natural substitute for boundary conditions on null infinity are boundary conditions at the cosmological horizon. We expand on the discussion of multiple horizons and complementarity given above. While this paper was being written we listened to E. Witten's talk at Strings 2001, using the magic of the Internet. There is some overlap with our considerations, and we devote section five to explaining our view of the relation between his talk and this paper. An appendix is devoted to a parable whose aim is to give philosophical solace to those who are disturbed by the notion of applying quantum mechanics to the universe.

[4] but also, we believe, exhibits some differences from

2. S-matrices in cosmology

2.1 Classical Considerations

Let us approach the construction of S-matrices for cosmology from the point of view of classical field theory. Morally speaking, what we are looking for are solutions with boundary conditions on a Big Bang singularity as well as the future null infinity of an eternally expanding FRW cosmology[5]. This means that, in analogy with the solutions with Feynman boundary conditions in asymptotically flat spacetime, we are looking for *complex* solutions of the field equations.

However, the problem is much more involved than that of asymptotically flat spacetimes. First of all, we are asking for boundary conditions at a singularity, so that the mathematical problem is ill posed. We can finesse this for the moment by asking for boundary conditions a few Planck times (in a synchronous coordinate system) after the Big Bang. More importantly, most initial conditions at an initial value surface near the Big Bang, will not be compatible with a future evolution of the universe which corresponds to a few particles (infinitesimal wave packets) propagating in an asymptotically FRW universe. Indeed, we see immediately that the problem cannot be formulated in a purely classical manner. That is, the insistence on a final state consisting of well separated freely propagating stable particles might be viewed as ruling out solutions in which the final state contained large black holes. However we know that, in the fullness of time, and when quantum mechanics is taken into account, such states will indeed decay into a finite number of stable particles[6]. To take this into account at the classical level we have to accept final configurations consisting of any finite number of finite mass black holes plus any finite number of stable particles, propagating in an expanding FRW geometry. We cannot know what other kinds of restrictions and caveats we must put on a classical treatment of the S-matrix, without solving hard problems like the Cosmic Censorship Conjecture.

The solution of this sort of boundary value problem is a highly nontrivial unsolved problem in classical general relativity. It is important to emphasize that it is very different from the initial value problem on a surface near the Big Bang singularity. Most initial conditions will not lead to an FRW universe containing only a finite number

[5]We will use the term FRW to refer not only to the standard homogeneous isotropic cosmologies, but also to anisotropic and possibly inhomogeneous cosmologies in which the spatial geometry is a compact Ricci flat manifold and may contain Wilson surfaces of p-form gauge fields. These are the most likely candidates for string/M theory examples of cosmologies with a future causal structure like that of the open and flat homogeneous cosmologies.

[6]Let us agree to work in more than four large dimensions, or simply to ignore the soft graviton infrared catastrophe.

of particles and finite mass black holes in addition to the homogeneous background. Instead they will have singularities. If Cosmic Censorship is valid, then perhaps all solutions will evolve to black holes plus asymptotic particles in the future. We find it equally likely that some subclass of classical initial conditions will have to be rejected because they lead to physically unacceptable naked singularities [7]. Nonetheless, the set of solutions of the classical equations which *do* satisfy these boundary conditions will define a sort of classical approximation to the Cosmological Scattering Matrix. Note that the Scattering matrix defined in this way will be invariant under diffeomorphisms which vanish at the boundaries of spacetime.

Similar considerations can be applied to classical string theory. Consider a space-time metric of the form

$$ds^2 = -dt^2 + R_i^2(t)(dx^i)^2, \tag{2.1}$$

in weakly coupled Type IIA or IIB string theory. When supplemented by a time dependent dilaton field these solve the lowest order beta function equations if the $R_i(t)$ take the familiar power law, Kasner, form. They may be viewed as the dimensional reduction of the Kasner solutions for toroidally compactified 11 dimensional SUGRA. There exist solutions of these equations which at large positive times approach a slowly varying, infinite volume torus with weak string coupling. The results of [13] show that the past asymptotics of these solutions is truly singular, in that there are no U-duality transformations which take them to weakly coupled string theories or to 11 dimensional SUGRA with slowly varying fields. Thus, in contrast to certain claims in the literature [14], string/M-theory appears to contain Big Bang cosmological singularities which cannot be removed by dualities. Despite the breakdown of string perturbation theory in these backgrounds, we can formally write down vertex operator correlation functions. At lowest order in α' the vertex operators are in one to one correspondence with solutions with given asymptotics on the boundaries of the spacetime, including the Big Bang. Of course, since both the α' and g_S approximations break down near the Big Bang, we are again left without a true definition of the scattering matrix.

2.2 Semiclassical considerations

Our finesse of the problem with the Big Bang Singularity was merely a stopgap measure. We believe that a true definition of a Cosmological S-matrix would have the following character: *The theory must simplify and become exactly soluble near the Big Bang, just as it does in the asymptotic future.* Then we could set up boundary conditions in terms of basis states of the Hilbert space which behave simply near the Big Bang. The

[7] Something similar has been claimed recently about singular solutions in the AdS/CFT correspondence [1].

Scattering matrix would be the amplitude to start in one of these simple basis states and end up in one of the simple states consisting of freely propagating FRW particles at null infinity.

We do not have the required initial solution in hand, but believe a few clues to its nature can be obtained by thinking about the Fischler-Susskind holographic bounds [12] . A more fundamental approach to the initial singularity will be sketched in the next section. Fischler and Susskind argued that the holographic bound was compatible with FRW cosmology if the equation of state of the cosmic fluid was no stiffer than $p = \rho$. With this equation of state the holographic bound will remain saturated for all times if it is saturated initially. For softer equations of state, the homogeneous entropy density can only account for a fraction of the holographic entropy at late times. Note also that the $p = \rho$ equation of state is the stiffest for which the velocity of sound is less than or equal to the velocity of light. If we accept these two indications that $\rho = p$ is the stiffest equation of state allowed by nature, and further assume that there is some matter with this equation of state, then we can conclude that it dominates the earliest stages of the universe. Its energy density scales like the inverse of the square of the spatial volume[8].

What can this primordial form of matter be? One possibility is the homogeneous modes of various massless fields. This includes massless minimally coupled scalars, topological modes of p-form gauge fields, and unimodular deformations of the space of Ricci-flat spatial metrics on a manifold with compact spatial sections. We will refer to these collectively as moduli. A problem with moduli from the point of view of the holographic principle is that they do not provide an entropy density but rather a finite entropy for the total spacetime. This would not appear to be a significant difference for a spacetime with closed spatial sections. The Fischler-Susskind description refers to a comoving entropy density, which, if the spatial manifold is closed, leads to a finite total entropy.

The important deficiency of moduli is revealed when one attempts to saturate the holographic bound. This leads to an equation of the form $\sigma_0 \propto \sqrt{\rho_0}$, where ρ_0 is the initial energy density of the system, and σ_0 its (constant) comoving entropy density. For moduli, ρ_0 is the initial energy of the homogeneous modes, divided by the volume of comoving coordinate space. It is easy to see that the entropy of moduli varies logarithmically with ρ_0. Instead, the formula suggests that in each Planck scale cell of comoving coordinate space there is a conformally invariant $1 + 1$ dimensional field theory (the CFT at the beginning of the universe). The states of these CFT's are

[8] We note, without claiming to understand the connection, that 't Hooft has used the $p = \rho$ equation of state in a model of black hole entropy[20].

constrained to have the same $L_0 + \bar{L_0}$ eigenvalue, but are otherwise independent of each other. It is not immediately apparent that such a CFT satisfies the equation of state $p = \rho$ in *spacetime*, since the mapping between its world volume and spacetime has not been specified. However, there is a general thermodynamic argument for homogeneous systems that the relation $\sigma \sim \rho^{1/2}$ implies the $p = \rho$ equation of state. In each comoving volume, the energy density, entropy density, temperature and pressure are related by

$$\rho = T\sigma - p. \tag{2.2}$$

This equation follows from the laws of thermodynamics ((2.3)) if we assume the energy and entropy densities, and pressure, are independent of the volume. Combining (2.2) with the general thermodynamic equation

$$dE = TdS - pdV, \tag{2.3}$$

applied to variations that leave the volume unchanged, and with the relation $\sigma = \alpha\rho^{1/2}$, we obtain

$$T = \frac{2}{\alpha}\rho^{1/2}. \tag{2.4}$$

Returning to (2.2) we find that $p = \rho$. Thus a model in which each comoving Planck volume has associated with it a $1 + 1$ dimensional CFT fits the phenomenology of a holographic universe and can saturate the holographic bound.

In fact, the requirement of saturation allows us to calculate the central charge of the conformal field theory in each Planck cell, if we normalize the length of the interval it lives on to 2π. Saturation of the holographic entropy bound requires that the entropy and energy in Planck units in a unit Planck volume, are related by $\sigma_1 = \frac{1}{4}(d-2)\sqrt{\rho_1}$.In the CFT the relation is $\sigma_1 = \sqrt{c\rho_1/6}$, where ρ_1 is the eigenvalue of $L_0 + \bar{L_0}$. The two formulae are identical if $c = \frac{3}{8}(d-2)^2$. We do not yet understand the significance of this result, which one would have hoped to be a clue to the nature of the CFT.

As noted in the introduction, our constraint that different cells have the same energy density is a solution of the horizon problem of cosmology. We note that this is not just putting in the answer by hand. Our considerations show that a homogeneous system can saturate the maximal entropy allowed by the holographic bound. This tells us that the initial conditions for the universe are much more highly constrained than local field theory would have led us to believe. In local field theory one can find homogeneous solutions of the equations of motion, but there appear to be a host of inhomogeneous perturbations of them. The holographic principle restricts the allowed perturbations because it bounds the entropy in regions of spacetimes which are close to the homogeneous cosmology. Furthermore, if $p = \rho$, and the initial entropy and

energy density are related in the proper manner (corresponding to the correct choice of central charge in the CFT model) then we have a homogeneous system that saturates the entropy bound. Thus there cannot be any nearby states which are inhomogeneous. Our decision to impose the constraint of equal energy for the CFT in each Planck cell was an attempt to model this fact about the holographic bounds, rather than to solve the horizon problem in an *ad hoc* manner.

We emphasize that this would be solution for the horizon problem will only make sense if we find a complete theory from which we can derive the phenomenological CFT model presented in this section. Further, there is no sense at the moment in which our considerations can be viewed as a replacement for inflationary cosmology. We do not have any evidence that our model solves any of the other cosmological conundra that are traditionally cited as evidence for inflation. Most importantly, we do not yet see a viable alternative model for the fluctuations which produce galaxies. In the next section, we will attempt to formulate a more basic approach to the dynamics of the Big Bang. Eventually, one would hope to derive the CFT model presented here from such a fundamental approach.

3. Prolegomena to a quantum theory of space-time

In this section we wish to present the beginning of an attempt to formulate Quantum Cosmology in a fundamental fashion. The formalism we will present is incomplete. It was discussed previously in the talk of one of the authors (TB) at the Strings at the Millennium Conference. The basic observation is that when the Fischler-Susskind-Bousso (FSB) bounds are applied to regions of spacetime consisting of the intersection of the causal pasts of a finite number of points, they give a holographic derivation of the concept of *particle horizon* in an expanding Big Bang universe. Since the notion of particle horizon is usually derived from locality, this is a clue to the relation between locality and holography. That relation has remained completely obscure in the Matrix Theory or AdS/CFT versions of holography. We argue that if we take the FSB entropy to be an actual count of the number of states, rather than just a bound, a picture of the Universe emerges in terms of a net of interlocking operator algebras. What is remarkable about this picture is that geometrical facts about a Lorentzian spacetime are translated into algebraic facts about quantum operator algebras. We argue that the quantum formalism can encode both the causal and metrical structure of the spacetime, and conjecture that (once a full set of axioms for the nets of algebras is discovered) there will, in the limit of large dimension algebras, be a unique spacetime corresponding to a given net of algebras.

A Lorentzian spacetime is characterized by its dimension, its causal structure and its conformal factor. The axioms we have formulated so far give information about the last two kinds of geometrical data, but not about the dimension. We sketch a program for understanding the spacetime dimension as well as the axioms we believe are missing. The idea is to find the algebraic structure corresponding to the $p = \rho$ state of the universe that we discussed above. Since this appears to characterize a situation in which, for a given spacetime dimension, we have the fastest rate of growth of information inside the horizon, we can hope to discover it by finding the fastest consistent growth rate of information in the algebraic formalism. Unfortunately, it is precisely the rules for consistent time evolution that we do not understand in the algebraic formalism. Thus, our discussion is just a sketch of a plan for discovering a fully consistent theory of quantum cosmology.

Consider a spacetime that begins with a Big Bang Singularity. That is, there is a spacelike surface on which the universe begins. Furthermore, the proper time between any point in the spacetime and the Big Bang surface is finite. Let us define a Past Intersection Region (PIRE) as the intersection of the causal pasts of a finite number of points. A PIRE which is the causal past of a single point will be called a basic PIRE. The boundary of a PIRE is an almost everywhere null region. Thus, there is an FSB bound for the entropy flowing through the boundary of any PIRE. This can be viewed as the maximum entropy that could be observed in any experiment done inside the PIRE. We will assume that this entropy bound is in fact saturated for the basic PIREs. R. Bousso has suggested to us that this cannot be so for generic PIREs, whose light sheets are artificially truncated by the surfaces of intersection. That is, we can associate to each basic PIRE a Hilbert space whose dimension is given by the exponential of the area of a holographic screen for the PIRE. Notice that all of these dimensions are finite, as a consequence of the finite proper time to the Big Bang. For generic PIREs we construct a Hilbert space whose dimension is the exponential of the area of the largest light cone that fits inside the PIRE.

Consider now some region of an expanding universe and two PIREs $P_1 \subset P_2$. It follows immediately from the holographic bounds that an observer in P_1 can observe fewer states than an observer in P_2 (equivalently, his observables constitute a smaller operator algebra) . This only makes sense if in fact the operator algebra A_1 is a tensor factor in A_2 *i.e.* $A_2 = A_1 \otimes \bar{A}_{12}$. Otherwise, observations in P_1 would not commute with observations in the part of P_2 disjoint from it, and there would no way to consistently discuss the *information in* P_1. The fact that a PIRE has an operator algebra that commutes with the algebras of other regions is usually derived from locality. In local field theory one assumes that every spacetime region has an operator algebra associated with it that commutes with the algebras of all other regions that are separated from it

by a spacelike interval. This principle of locality is not compatible with the holographic principle.

However, we see that the holographic principle itself forces us to the concept of *particle horizon*, which we usually derive from locality. Each PIRE has only a finite number of states associated with it and smaller PIREs have a smaller number of states. One can dimly glimpse how our conventional concepts of local evolution may be compatible with holography.

Thus, we propose to associate a finite dimensional Hilbert space to every PIRE. Furthermore, every pair of PIREs has an intersection (possibly empty) which is also a PIRE. Thus if H_I and H_J are the Hilbert spaces of two PIREs then we must have

$$H_I = H_{IJ} \otimes D_{IJ}, \tag{3.1}$$

$$H_J = H_{JI} \otimes D_{JI}, \tag{3.2}$$

$$U_{JI}^{-1} = U_{IJ} : H_{IJ} \to H_{JI}. \tag{3.3}$$

That is, each Hilbert space must have a tensor factor which is shared between the two and represents the Hilbert space of the intersection. The invertible unitary isomorphism U_{IJ} takes into account the possibility of a different choice of basis made by observers in each PIRE for this common Hilbert space. The relations between members of this collection of Hilbert spaces encode information about the causal structure of the spacetime, and their dimensions tell us something about its geometry.

Now let us consider a network of basic PIREs, the tips of whose light cones are only a Planck distance apart along spacelike or timelike geodesics. Note that such a network inevitably involves some coordinate choices: there are many inequivalent ways of choosing a lattice of Planck separated points in a spacetime. We call any such choice, a *Planck lattice*. It seems clear that the net of PIREs associated with a Planck lattice, along with the areas of their holographic screens, should determine the geometry of spacetime, with at least Planck scale accuracy. Since the information about the PIREs, their overlaps, and their holographic areas, can all be encoded into properties of a net of Hilbert spaces, we will propose that quantum spacetimes are simply such a net, obeying appropriate axioms.

One final point before proceeding to this program. If we have two PIREs such that $P_1 \subset P_2$ then it is clear that $H_1 = H_{12}$. However, the converse is not necessarily true. Consider for example an FRW universe with vanishing cosmological constant and a black hole embedded in it. If we examine the causal past of a point inside the horizon of the black hole, the holographic screen of this PIRE lies entirely outside the horizon and can be completely contained in the causal past of points in the asymptotic region. In this case we would also say $H_{BH} = H_{BH,\infty}$ even though part of the black

hole PIRE is causally disconnected from the outside. The inclusion relations between Hilbert spaces reflect those between holographic screens, rather than spacetime regions. This rule is necessary in order to construct a formalism, which does not have a black hole information paradox.

3.1 Geometry from quantum mechanics

Now we would like to turn these relations around and propose a set of axioms for nets of Hilbert spaces[9] that will allow us to reconstruct a spacetime. We will be only partially successful in this endeavor. We begin with a list of a countably infinite set of finite dimensional Hilbert spaces H_n. Each of these are supposed to represent the quantum observables in the causal past of a single point. We emphasize however that the quantum mechanics is fundamental, while the geometrical interpretation of it is only supposed to emerge in a limit of very large dimension Hilbert spaces. For each pair of Hilbert spaces we have a set of equations analogous to (3.1):

$$H_m = H_{mn} \otimes D_{mn}, \tag{3.4}$$

$$H_n = H_{nm} \otimes D_{nm}, \tag{3.5}$$

$$U_{nm}^{-1} = U_{mn} : H_{mn} \to H_{nm}. \tag{3.6}$$

Now we add the H_{mn} to our list and repeat the procedure. However, we must also add rules which assure that *e.g.* $H_{mn,k} = H_{m,nk} = H_{k,nm}$ *etc.*. All of these spaces must have the same dimension and there must be unitary mappings between them which are compatible with the unitary embeddings of single overlaps into their parent H_i. Thus there will be a single Hilbert space H_{ijk}, symmetric in all indices, which can be embedded as a tensor factor of each Hilbert space with one or two of the indices (i, j, k). In a similar manner we can build k fold overlaps $H_{i_1 \ldots i_k}$. We will consider a set of multiply indexed Hilbert spaces satisfying the above tensor inclusion relations (plus some other axioms that we do not yet understand) to be the quantum version of a cosmological spacetime.

We view the above construction as the analog of constructing an atlas of charts which covers a Lorentzian manifold. These charts contain information both about causal structure and metrical geometry. The former is encoded in the inclusion relations between Hilbert spaces, which should be mapped into the causal relations between holographic screens. The dimensions of the Hilbert spaces encode metrical information. One would like to prove that in the limit of large dimensions, such a Hilbert space construction in fact determined a unique Lorentzian spacetime. We believe that there

[9]Equivalently, since we are dealing with a finite dimensional situation, for nets of operator algebras.

are a number of axioms which must be added to our system before one can hope to prove such a theorem. We will describe a number of different problems that we do not know how to solve, but they may not all be independent.

The first set of problems has to do with the analogy to the definition of a manifold. There one starts from an abstractly defined topological space that satisfies the Hausdorff property that any two points can be separated by open sets. Our holographic geometries do not have points and it is not clear to us what the analog of the Hausdorff property is. Furthermore, the abstract topological space in the definition of a manifold allows us to define the idea that an atlas covers the manifold. In what we have done so far the charts themselves seem to define the manifold. We believe that this is a symptom of a very deep property of the holographic approach to spacetime: one cannot decide whether two regions of spacetime are independent of each other (have independent degrees of freedom) until one knows their ultimate fate. Thus, a lengthy inflationary period followed by reheating and a matter or radiation dominated FRW cosmology, looks like an AsDS space for many e-foldings. If the DS era lasts forever, so that the cosmological horizon is a true horizon, then we consider the degrees of freedom in different horizon volumes to be gauge copies of each other. The experience of different causally disconnected observers is thought of as arising from a different choice of basis in the same Hilbert space. On the other hand, in the inflationary cosmology we consider these degrees of freedom as independent (and use them as a basis for calculating cosmic microwave background fluctuations) because an observer in the far future of the inflationary era will be able to measure correlations between them.

Another example that illustrates the same problem can be constructed by thinking about a Big Bang Universe which asymptotes to DS space. As in the previous section, and according to standard convention, one describes the early universe in comoving coordinates and assigns it a homogeneous entropy density. If the equation of state at early times is $p = \rho$ then such a picture can capture the correct holographic counting of entropy at arbitrarily early times, bounded from below only by our decision about when the semiclassical picture breaks down. Furthermore, assuming compact spatial sections, there is nothing in the FSB discussion that restricts the comoving coordinate volume. Thus, we could assign such a universe an arbitrarily large total entropy, using conventional language. On the other hand, one can construct models in which such a universe asymptotes to a DS space with DS entropy less than the total entropy of the universe at early times. There is no contradiction here because this simply means that many different coordinate volumes will be outside each other's particle horizon forever. The holographic principle again tells us to treat these degrees of freedom in terms of different bases in the same Hilbert space, rather than independent Hilbert spaces. But there is no way to make this judgement without knowing the future asymptotics of

the cosmology. This makes sense when thinking about covering a Lorentzian manifold by PIREs, but is at odds with the usual notion of physics being determined by initial conditions. It is more compatible with the Feynman propagator or S-matrix approach, in which boundary conditions on both past and future are necessary to completely specify a physical process.

This brings us to the most serious difficulty with our formalism, which is how to construct a unitary time evolution operator. We should recognize from the beginning that their should be no unique prescription for such an operator. Different physical coordinate systems should have different evolution operators. In the semiclassical approach to quantum gravity, time evolution is defined in terms of certain approximately classical variables[15]. These can be viewed as defining a classical background geometry. Even in this situation there can be various natural definitions of time which are not related by isometries.

Quantum mechanics is usually described in terms of a Hilbert space and a Hamiltonian operator. It is important to recognize that the Hilbert space tells us almost nothing about the system. Any two separable Hilbert spaces of the same dimension are unitarily isomorphic. It is the Hamiltonian that defines what we usually think of as "the structure of the Hilbert space of the system". For example, many Hamiltonians have the property that their asymptotic high energy spectra are identical to those of a system of some number of Gaussian variables. One can describe any Hamiltonian with the same Gaussian fixed point in terms of differential operators acting on wave function(al)s of the Gaussian variables. This is the conventional formalism of Schrodinger quantum mechanics. In a system without an *a priori* Hamiltonian one might imagine *any* one parameter group of unitaries (or perhaps just a discrete unitary group if we give up the idea of continuous time evolution- see below) be viewed as time evolution for some observer. It is only in a regime where the system has many states and enough variables to behave classically that one could dismiss many of these would be observers as "unphysical". Unphysical evolutions would be defined to be those which do not preserve the classical nature of the classical variables.

In our formalism, the Hilbert space itself has much more structure. The whole system can be mapped into a single Hilbert space, the space of the asymptotic future[10]. This is defined as a Hilbert space H_∞ such that any of the H_n are tensor factors of it. Furthermore, we insist (as an additional axiom if necessary) that there is some sequence of H_{n_i} which converges to H_n. However, any definition of an evolution operator on H_∞ must be compatible with the causal structure that is implicitly defined by the $H_{n_1 \ldots n_k}$.

[10]We restrict attention to cosmologies that expand indefinitely. The Big Crunch requires a separate discussion.

Let us examine some of the consistency conditions that a time evolution must satisfy. First, it must "foliate" the collection of Hilbert spaces , $H_{n_1 \ldots n_k}$. That is, it must break the collection up into subsets, each of which is "on a given time slice". We denote this by giving each Hilbert space a label t. Thus, the indices n of the previous discussion are broken into a composite index (t, n), where we abuse the reader's patience by using the same letter for the "spatial" part of the composite index as we previously used for the whole. The slicing must be compatible with the existing causal structure. Thus, every $H^{t_1 \ldots t_k}_{n_1 \ldots n_k}$ with all $t_i \leq t$ should be a tensor factor of some H^{t+1}_n. The real problem is to find a map which gives us a unique state in each of the Hilbert spaces whose largest time index is $t + 1$ given a state in each of the Hilbert spaces with largest index t. It is plausible that if it is possible to find time slices for which such a map exists, that there will be many inequivalent ways to do so. This would correspond to the many fingered time of general relativity. However, it is clear to us neither whether it is always possible to find such a time slicing, nor what axioms have to be added to the structure of our net of Hilbert spaces in order to guarantee that such a slicing exists. Another puzzle is the question of the uniqueness of the map between successive slices and what the proper interpretation would be for finding many consistent maps. At the present we do not have an answer to any of these questions, which explains the title of this section.

The reader will have noted that the time evolution we describe is discrete. This seems to follow inevitably from the idea that time evolution always follows expansion of the size of the particle horizon plus the fact that area is quantized in bits. We should identify the time step with the Planck size time steps in the lattice of basic PIREs described in the previous section. Indeed, it seems that the right way to understand the problem of time evolution is to try to construct nets of Hilbert spaces which satisfy the inclusion relations of the Planck spaced web of PIREs that we constructed in the previous section. We will reserve this project for a future paper, apart from the remark that this construction makes it clear that our nets of Hilbert spaces represent a *gauge fixed* construction of the theory. We could describe the same spacetime by constructing the net of Hilbert spaces of many different Planck lattices. These are all *physical* gauges , in the sense that all of the Hilbert spaces have positive definite metric. They should represent the physical measurements of observers who choose different coordinates to describe spacetime (and have also made choices of holographic screen that are locked to the coordinate choice), whenever the evolution is sufficiently classical to justify the separation of the system into an observer and the rest of the universe. Nonetheless, none of this information is strictly gauge invariant. We reserve to the future the important task of formulating the equivalence relation between nets of Hilbert spaces implied by general coordinate invariance, as well as the additional axioms that will guarantee that a net of Hilbert spaces can be realized as the net of PIREs associated with a Planck

lattice in a spacetime satisfying the dominant energy condition.

Another very interesting question, about which we have only conjectures, is the extent to which information about the universe is encoded purely in the net of Hilbert spaces rather than the choice of state in these Hilbert spaces. One might imagine that consistency conditions of some sort were strong enough to completely specify the state [11]. We believe that it is more likely that in the semiclassical limit there will be a sense in which the net of Hilbert spaces contains information only about the geometry, while the state will tell us about the properties of branes propagating in the geometry. As a consequence of M-theory dualities, even this cannot be exactly right under all circumstances. But such ambiguities really only arise when the universe has small dimensions and neither of the U-dual notions of geometry is valid. As all dimensions become large, the separation between geometry and branes becomes sharp. The property that not all the information in our systems is in the structure of the net of Hilbert spaces is shared by our $1+1$ CFT model of the early universe. There, the geometry is determined by and determines only the $L_0 + \bar{L}_0$ eigenvalue of the CFT, but not its state.

Finally, we can return to our discussion of the S-matrix. If our ideas about Planck lattices are correct, it will be very easy to impose the condition that the net of Hilbert spaces asymptotically approaches that of an eternally expanding FRW universe with a boundary causally equivalent to that of Minkowski space. The advantage of our present formalism is that we can make coherent remarks about the initial state. Indeed, it is obvious that as we go back in time, the particle horizon becomes smaller and smaller. At some point, its area in Planck units becomes of order 4ln2. The dimension of any Hilbert space on this slice of comoving coordinates is no bigger than two. We cannot extrapolate the evolution back any further. This is the point we wish to identify with the Big Bang. The quantum state of the universe is a tensor product of independent states in a collection of two state systems. There are no more overlaps, because 2 is a prime number[12]. In an expanding universe with vanishing cosmological constant the number of two state systems will be infinite.

Note that in making these statements we have not had to specify any particular properties of the net of Hilbert spaces. Thus this initial state will be gauge invariant under the equivalences that we have discussed above. The detailed properties of the spacetime are encoded in the evolution rules which generate the rest of the net. These, as we have discussed, are gauge variant. By imposing asymptotic conditions on the net

[11] We owe this radical conjecture to R. Bousso.

[12] D.Gross and L.Motl pointed out to us that any other prime might do as well, and that one could imagine different primes in different initial Hilbert spaces. This issue will have to be confronted if we are to make believable claims about noninflationary solutions to the horizon problem.

in the infinite future, we obtain a set of completely gauge invariant amplitudes. The different but equivalent nets will represent different asymptotic coordinatizations of the system, but, in the conventional manner, these are not imposed as gauge symmetries[13]. If the eventual fate of the universe were indeed an FRW spacetime, we could choose to restrict our attention to nets that approached those defined by Planck lattices in comoving coordinates.

There is a further curious possibility that our ignorance of the full set of axioms prevents us from making a definitive statement about. Recall that the embeddings of earlier Hilbert spaces as tensor factors of later ones involve unitary mappings. Then it may be that the choice of initial tensor product state is a gauge choice, *i.e.* that the initial state is unique up to gauge transformations. If this is the case, then all information about which universe one was describing, would be encoded in the laws of time evolution. This would mean that the problem of time would be mixed up with what is generally called the question of initial conditions.

Thus, we believe that our formalism will be able to give a completely well defined and gauge invariant definition of an S-matrix for FRW spacetimes with vanishing Λ. The initial state is a tensor product state of an infinite collection of two state systems. The final state consists of a finite number of stable particles propagating on an FRW spacetime. The tensor product state of two state systems does not bear any resemblance to a spacetime. The conventional semiclassical regime will set in only after propagation to the point where the Hilbert spaces in our net define areas large compared to the Planck scale. It is in the early stages of the semiclassical era that one can expect the $p = \rho$ phase to occur. It seems to be the simplest semiclassical cosmological era and we hope to encode its properties in the rules for evolving the net. This project as well will be reserved for a future paper.

To summarize this meandering section: It is clear that the set of PIRES (and their holographic areas) generated by a Planck lattice contains enough information to determine the spacetime in which they are embedded. Since all this information can be translated , via the holographic principle, into information about a net of interlocking Hilbert spaces (operator algebras), we have a strong indication that a set of axioms formulated solely in terms of such Hilbert nets can reproduce classical general relativity as an approximation. We have formulated some, but (we are quite sure) not all, of the axioms. The formalism promises to give a picture of black hole formation and decay which is manifestly free of paradoxes, and incorporates the Black Hole Complementarity principle. It gives a picture of the Big Bang as a collection of decoupled two state

[13]Usually we only discuss the asymptotic isometries of *e.g.* asymptotically flat spaces, but the whole set of coordinate transformations that act nontrivially on the asymptotic space can be viewed as physical operations on the Hilbert space of the system.

systems. The gauge invariances of the formalism may imply that there is a unique initial state of this system. We hope that by studying the transition of this system into a quantum model of $p = \rho$ cosmology, we will be able to learn the rules for constructing a general quantum geometry.

4. Gauge variant S-matrices for AsDS spacetime

We now want to generalize our considerations to the case of AsDS spacetimes which begin with a Big Bang singularity. In our opinion, the proper generalization is a matrix which interpolates between states at the Big Bang (which we now recognize as tensor product states in a collection of two state systems) and states on the cosmological horizon. As we take Λ to zero, a single horizon volume in DS space approaches all of Minkowski space, with the horizon approaching the boundary of Minkowski space. A similar statement is valid for AsDS cosmologies and FRW cosmologies with vanishing Λ. What is confusing, is that there are an infinite number of different cosmological horizons in AsDS space. So our prescription has an ambiguity. Furthermore, the different horizon volumes are mapped into each other by diffeomorphisms so no given S-matrix is gauge invariant.

We believe that this correctly represents fundamental physical properties of AsDS spaces[14]. The key to understanding this is the Black Hole Complementarity Principle of 't Hooft, Susskind, Thorlacius and Uglum [18], and we will begin by briefly reviewing these arguments. Let us remind the reader that although a plausible case has been made that scattering off a black hole is unitary, we have as yet no clues about how to describe the experience of the infalling observer. The BHCP is a slogan that outlines what such a description might look like. We will summarize the arguments of [18] by the statement that thought experiments show that no comparison between the states of the infalling and external observers in a black hole is possible within the realm of low energy effective field theory (which is the only realm in which the concept of infalling observer is clearly defined). HSUT argue that this means that the claim that the external observer's Hilbert space of scattering states is complete is not ruled out. In such a description, the infalling observer makes measurements in the same Hilbert space, but measures observables which are complementary to those of the external observer. One cannot compare the measurements because they interfere with each other. Since the infalling observer's time evolution is finite, the only precisely defined gauge invariant observable of the system is the external S-matrix. The quantum mechanical definition

[14] which L. Susskind has characterized as "the great crisis in theoretical physics that would be caused by the observational proof that there is a positive cosmological constant". As will be clear below, we take a somewhat more sanguine view of the situation.

of the infalling observer is neither precise nor gauge invariant, except perhaps in the formal limit of infinite black hole mass.

Now consider the situation in an AsDS space. For any given observer, all things that are not bound to her appear to be squeezed closer and closer to her horizon volume as the Hubble flow sweeps them away. All that remains of them is a thermal gas of Hawking radiation. If the observer is part of a large, gravitationally bound system, then she is likely to eventually end up as an infalling observer for a large black hole. After the black hole decays, its remnants are swept into the Hawking gas clumped near the DS horizon. All that remains is a complementary (but decidedly uncomplimentary) image of the Hilbert space states that once described her existence. If she has cleverly avoided this fate by building a large steel living module somewhere in the space between galactic clusters, her agony is only prolonged. The Second Law of Thermodynamics assures us that she will eventually be thermalized and become part of the Hawking DeSitter gas. Thus, *all* observers in AsDS space, regardless of sex, race creed or color, are analogous to infalling observers for a large black hole. The true final state of the system is always one of the states of the thermal ensemble.

The thermal ensemble is DS invariant and therefore gauge invariant. Furthermore, once we agree that, asymptotically, there are no macroscopic objects left in DS space, one might imagine that the only sense in which individual states that make up this ensemble fail to be gauge invariant is that they are mapped into identical states in another horizon volume. These would then have a gauge invariant definition as well. The question of the nature of these states depends on details of the full theory of quantum gravity that we do not yet understand. The microscopic theory of these states might show that the natural basis for describing them was one in which the observables describing finite macroscopic objects localized in DS space were not diagonal. One could then imagine constructing identical bases in the Hilbert spaces of different horizon volumes and thus a completely gauge invariant S-matrix for AsDS systems. But if the S-matrices for different horizon volumes were identical they could contain no information about the existence of macroscopic observers with different experiences in different horizon volumes. So, either there is no gauge invariant S-matrix, or it does not contain information about the physics that we think is interesting.

The difficulties inherent in describing gauge invariant DS physics, are closely connected with the *problem of time* of traditional quantum cosmology. There are no preferred unitary operators in a cosmological Hilbert space except in extreme circumstances where semiclassical approximations are valid[15]. We have already remarked

[15]and, if our conjectures are correct, very near the Big Bang singularity, where we hope that the physics becomes soluble, though not classical.

above that *all Hilbert spaces look the same in the dark.* Without the guide provided by a Hamiltonian, we cannot distinguish different systems with the same number of states. Thus, without a compelling set of semiclassical observables, there is no reason to prefer one sequence of unitary transformations in Hilbert space from another. Our causal nets of Hilbert spaces provide us with a little more structure, but at least to some extent this corresponds to a choice of coordinates. AsDS space and black hole physics present us with an even more disturbing situation in which there may be several different semiclassical descriptions, that are complementary in the sense that their time evolution operators do not commute with each other (this would be the mathematical statement of the BHCP). In the case of black holes, the external observer's semiclassical description is preferred, because it is completely well defined, while the infalling observer has only a finite lifetime and cannot be expected to have a complete and exact description of physics. In AsDS space, all macroscopic observers are equal, and equally ill defined.

Our interpretation of the apparent lack of gauge invariance of observables in AsDS spaces can thus be phrased as a DS Complementarity Principle. More generally, we interpret the Problem of Time as a Cosmological Complementarity Principle. That is, different physical observers in a Cosmological spacetime will generically have time evolution operators that do not commute with each other. The Complementarity principle tells us that these operators all act in the same Hilbert space, even when we are referring to two sets of observations that are out of causal contact forever. In some cases, there will only be a single set of semiclassical observables and we can choose these to define special classes of gauge transformations under which the physical Hilbert space is not required to be gauge invariant. The asymptotic observables in various kinds of asymptotically infinite spacetimes are a particular example. AsDS spaces are different because there are in principle an infinite number of equally good semiclassical time evolution operators at late times.

The difficulties of interpretation of AsDS physics are also connected to the finite dimension of the Hilbert space that represents AsDS space[3]. We suspect that for Hilbert spaces with dimension 2^N with N which is not enormously large, there will be only a few causal nets of Hilbert spaces that can be constructed. None will have a spacetime interpretation and no consistent unitary evolution will be preferred over any other. There is unlikely to be gauge invariant physics associated with such a system. In a universe with only a few states, a basic assumption of all physical theories[16], becomes untenable. This is the claim that one can separate the system into *observer* and *observed* with sufficiently small interaction between them that one can make a

[16] not just quantum mechanics

measurement without completely changing the property one was trying to measure, or destroying the measuring apparatus. As N becomes large the possibility of forming large classical subsystems emerges. We have argued that all such subsystems have finite lifetime. However, we should remember that if our universe has a cosmological constant, the value of N is 10^{123} . The finite lifetime for classical subsystems is of order the infall time for all galaxies to collapse completely into black holes (for gravitationally bound systems) or of order the thermalization time (for isolated space platforms). We note that the average number of Hawking particles in the Hawking-DeSitter gas is one per horizon volume[17] , so the thermalization time is unimaginably long. Thus we believe that for very small Λ it will make sense to restrict the gauge group to gauge transformations which leave a particular horizon volume invariant. The other gauge transformations will be viewed as physical operations, which act on the Hilbert space of the system[18] . Thus, we claim that for very large N, one can already begin to distinguish the special observables of the $N \to \infty$ boundary theory. In a sense, the large classical subsystems in a given horizon volume are acting like a Higgs field which picks out a particular gauge frame.

There will clearly be corrections to such an approximate picture. However, not all corrections will disturb the approximate treatment of large subsystems by conventional quantum mechanics. We conjecture that as long as one does measurements that involve only a small fraction of the entropy of the universe, one does not have to worry about the conceptual issues of working with a finite system and doing measurements over finite time intervals. Only if one wanted to enquire into the nature of the exact quantum state of the Hawking DeSitter gas would one find oneself performing operations that were physically ambiguous. Since the Hawking temperature is very low, the uncertainty introduced into ordinary measurements by our lack of knowledge of the state of the horizon is exponentially suppressed as $N \to \infty$.

5. Relation to a proposal of Witten

In his talk at Strings 2001 [16], E. Witten proposed observables for spacetimes that are AsDS in the past, or the future, or both. His proposal was described in the standard

[17]These most probable components of the radiation also have wavelengths of order the horizon volume and have little effect on localized objects. The probability of finding dangerous shorter wavelength radiation is even smaller.

[18]Indeed, as we have defined the Hilbert space H_∞, in the previous section, all gauge transformations under consideration act on it. The distinction is between unitary maps which leave a given causal net of spaces embedded in H_∞ invariant, and those which do not.

global coordinate system for DeSitter space

$$ds^2 = -dt^2 + R^2 \cosh^2(t/R) d\Omega^2, \tag{5.1}$$

in which the manifold is realized as a $d-1$ sphere that contracts from infinite radius to a finite radius R and reexpands to infinity, as t ranges over $[-\infty, \infty]$. The boundary of this manifold is the union of the infinite spheres at past and future infinity. Witten proposes to define asymptotic states on these two boundaries (or on the union of the DS boundary at future infinity, and a Big Bang singularity in the past, for the case of an AsDS Big Bang cosmology). The attraction of this proposal is that it appears to define an S-matrix that is invariant under all diffeomorphisms of an AsDS spacetime. Witten claims that semiclassical analysis of this proposal leads to the conclusion that there are an infinite number of asymptotic states. He argues that this could be compatible with the claim of a finite number of states for AsDS spaces if the S-matrix he defines actually has finite rank in the infinite dimensional space of asymptotic states.

Let us first assume that Witten's analysis is correct and inquire into the interpretation of this finite rank matrix. It appears to us that the procedure of modding out by the zero modes does not lead to a unique S-matrix. Unlike the case of BRST quantized gauge theories, the metric on the space of asymptotic states is positive definite (Witten claims to be analyzing diffeomorphism invariant physical states of quantum gravity perturbatively quantized around a DS background). Thus, it is impossible for different versions of the finite dimensional S-matrix to be equivalent to each other. There is no procedure for extracting it which is invariant under unitary transformations in the space of asymptotic states. That is, even if, using some conventions, we define a basis in which Witten's S-matrix is block diagonal, with one block being a finite dimensional unitary matrix S^N_W and the others vanishing, we can, by performing a unitary transformation in the big space, turn this into $V S^N_W W$, so that the finite dimensional S-matrix is ambiguous. If this is the case, then we suspect that the ambiguity will turn out to be equivalent to the one arising in our prescription from the absence of an invariant way to choose a particular cosmological horizon. We have already explained why we think that this ambiguity actually reflects the correct physics of DS backgrounds via the DS Complementarity Principle.

However, we also believe that the evidence for Witten's claim that the theory has an infinite number of states in the semiclassical approximation is less than compelling. One way to semiclassically quantize General Relativity in DS backgrounds is by analytic continuation of semiclassical Euclidean path integrals on the sphere. This analysis was presented in [3]. General results of quantum field theory in curved spacetime relate these path integrals to gauge fixed free massless spin two fields, plus matter fields in the static patch of some particular cosmological horizon [17]. Furthermore, the

gauge fixing procedure instructs us to integrate over the DS group, which is part of the group of diffeomorphisms. Since the DS group maps every horizon patch into every other one, we are instructed to treat all information outside the horizon as a gauge copy of information inside the horizon. This is a strong argument in favor of the DSCP. The result of Euclidean functional integrals is the thermal state for the timelike Killing vector of the static patch. We can construct other (impure) states by analytically continuing correlation functions of the form $< O^\dagger(+)\phi(z_1)...\phi(z_n)O(-) >$, where we have used the gauge fixing procedure to place a BRST invariant operator and its conjugate at the north and south poles of the coordinate we will analytically continue to Lorentzian time. These give density matrices of the form $O^\dagger\rho O$, where ρ is the thermal state. By construction, each of these gives rise to a DS invariant state in the sense of Witten. It is not clear to us whether all of the states described by Witten can be constructed in this way, but it would seem odd to find the correspondence between Euclidean and Lorentzian signature quantum field theory breaking down at this level. Assuming the two constructions are equivalent, we can inquire into the origin of the apparent infinite number of states in the construction. Since the static patch of DS space has finite volume, it is clear that this is a UV infinity. Thus, from the point of view of the static patch, the infinite number of states comes from the region where we do not trust the semiclassical approximation.

How is this compatible with the analysis in the global coordinates (5.1)? There the infinity can be viewed as coming from infinite numbers of well separated wave packets on the infinite radius spheres in the future and/or past. Since the spheres have infinite radius, we do not have to go to asymptotically high energy to localize excitations. So here we seem to establish the existence of an infinite number of states without invoking UV degrees of freedom for which the semiclassical approximation breaks down. We believe the key to understanding the consistency of the two analyses comes again from the singularity theorems of General Relativity. That is, we believe that initial conditions in global coordinates which appear to violate the Bekenstein-Hawking bound for DS space will lead to solutions with singularities. The physical mechanism for this is that the background DS evolution squeezes all matter into a finite radius R at global time $t = 0$. Thus, even very dilute matter on the sphere at infinity will evolve into (must have come from in the case of future infinity) matter whose gravitational field cannot be neglected.

There appear to be two possible interpretations of these singularities. Some form of Cosmic censorship might be valid in AsDS spacetimes, in which case these initial conditions could be classified in terms of black holes in DS space. They would then have only finite entropy. On the other hand, some or all of the initial conditions violating the bound might have to be thrown away because they produced singularities that

were unacceptable (*cf.* [1]).Again, the question of infinite numbers of physical states would seem to depend on the behavior of the theory in regimes where the semiclassical approximation breaks down. In our opinion, the most likely conclusion is that the system has only a finite number of physical states and a truly gauge invariant S-matrix does not exist. This reflects the semiclassically verifiable conclusion that observers in different horizon patches will see different semiclassical physics, combined with the DS Complementarity Principle, which asserts that all of these observations are measuring operators defined in a single finite dimensional Hilbert space. Of course, it is only in the limit of very large dimension (very small cosmological constant) that we expect any of the physics to have a semiclassical description.

6. Discussion - the relation to string theory

What is the relation of all of this to string theory or M-theory (which we use in the sense of the theory underlying the various semiclassical expansions embodied in perturbative string theory and 11D SUGRA)? We believe that M-theory has various incarnations which, loosely speaking, depend on asymptotic boundary conditions in spacetime. There is no background independence in the sense that most string theorists have assumed in recent years. That is , not all of the versions of M-theory can be thought of as different representations of the same operator algebra in Hilbert space[19] . M-theory in asymptotically AdS spaces is described quantum mechanically by conformal quantum field theory. M-theory in asymptotically flat spaces is described by some as yet undiscovered quantum operator algebra which naturally reproduces the high energy black hole spectrum [20]. M-theory in linear dilaton backgrounds is Little String Theory. M-theory in $\Lambda = 0$ FRW spacetimes may have a a description in terms of the same operator algebra as that of asymptotically flat spaces, but also requires a dual description in terms of simple operators at the Big Bang. The gauge invariant information in this theory is encoded in the S-matrix between these two descriptions. M-theory in AsDS spaces lives in a finite dimensional Hilbert space, *etc.*.

Our most far reaching conjecture would be that somehow all of these different versions of M-theory could be realized in terms of a set of axioms for nets of Hilbert spaces of the type we have discussed here. The different classes of M-theory would correspond to different asymptotic conditions on the net. Our ability to encode spacetime geometry in terms of Hilbert spaces associated with a Planck lattice makes us optimistic that

[19]This is the most general way to phrase what we mean by different vacuum states of the same theory in quantum field theory.

[20]In certain cases it can in principle be constructed as the large N limit of Matrix Theory.

such a construction will be possible. What is missing is a formulation of the dynamical laws directly in terms of the net of Hilbert spaces.

This philosophy leads us to be skeptical of attempts to find *string theory models of DS space*. If we are correct, both asymptotically flat string theory, and DS space, will be realized as special cases of the same underlying laws, but string theory will have too many degrees of freedom to describe DS space. An interesting possibility[21] is the construction of a metastable DS vacuum in models that are *e.g.* weakly coupled string theory in something like asymptotically flat space. Then the conventional stringy S-matrix might contain information about the DS "resonance". If Newton's constant is asymptotically constant, this attempt is likely to fail and the putative DS vacuum will remain forever shrouded behind a black hole [19]. However, in models where Newton's constant asymptotically rolls to zero, some progress might be made.

7. Appendix - quantum mechanics and cosmology

The idea of using quantum mechanics to describe the entire universe has been known to inspire unease in the breasts of some of our most eminent physicists. The basic problem is that the physical interpretation of quantum mechanics appears to depend crucially on the ideas of probability theory, and the operational definition of probability requires us to imagine the possibility of doing a measurement an infinite number of times under exactly equivalent conditions. On the other hand, the evolution of the universe occurs only once. Furthermore, the concept of measurement requires us to separate the universe into system and apparatus, and this may not be possible, even in principle, under all cosmological conditions.

We believe that there is a formulation of the principles of quantum mechanics which ameliorates this philosophical distress, without perhaps removing it entirely. It is essentially the Quantum Logic interpretation of Quantum Theory proposed by Von Neumann (and immediately dismissed by Bohr as a mathematical irrelevancy). Here we will present a brief review of this interpretation with a few linguistic twists. The essential observation is that classical logic can be reformulated in terms of a C^*-algebra [22]. Any question about a physical system can be turned into a yes no question. Thus, the statement that a certain variable has the value 5.2 is equivalent to the questions of whether or not it takes on any one of its allowed values, and whether 5.2 is among them. It is well known that the logical relations between any finite number of yes/no questions are equivalent to the (Boolean) algebraic relations between a maximal set of

[21] Suggested to us by E.Silverstein.

[22] For simplicity of exposition, we will imagine a physical system with only a finite number of states, so we really mean just a finite dimensional matrix algebra.

commuting projections in a finite dimensional Hilbert space. It is further obvious that we want to define a general observable in such a system as a real linear combination of the projection operators. We introduce complex linear combinations for the purely mathematical convenience of being able to solve general algebraic equations involving observables (it would be more satisfying to have a physical motivation for introducing complex numbers).

Thus, thinking purely classically, a physical system is related to the Hermitian elements of a C^*-algebra of operators in a Hilbert space. The quantum logician then makes the observation that this classical logic has chosen a special basis in the Hilbert space, or equivalently a special maximal abelian subalgebra. Being mathematically minded, she asks what the choice of a state of the system (a choice of the answers to all of the classical logician's yes/no questions) implies for all of the other Hermitian operators which do not commute with the special abelian subalgebra. She quickly realizes that each such state defines a probability distribution for any other maximal abelian subalgebra, and further that any such probability distribution for a given maximal abelian subalgebra is equivalent to a choice of classical state for *some other* maximal abelian subalgebra. Our quantum logician is one of those fortunate people who has both mathematical and physical intuition and invents the notion of *complementary observations.* That is she declares that all of the Hermitian operators in the Hilbert space represent possible observations on the physical system but that some measurements interfere with the results of others.

In particular, one must contemplate the change of the state of the physical system with time, an operation which, *even for the classical logician* is described by a unitary operator which does not commute with with his favored abelian subalgebra. The quantum logician identifies Hermitian functions of this unitary operator as candidate observables which will be complementary to the classical logicians preferred measurements.

If one can imagine an alternative history in which Boolean logic and the theory of operator algebras was developed previous to the invention of classical mechanics, one can imagine the seventeenth century quantum logician pondering the question of whether some of Mr. Newton's observable quantities might be complementary to each other. Presuming her to be very long lived, and noting that, as one of the few women in theoretical physics at the time, she would have been particularly sensitive to E. Noether's famous result connecting symmetries (particular operations on or changes of state) of classical mechanical systems with conservation laws (particular observables), one can imagine our quantum logician jumping to the absurd conclusion that position and momentum were complementary variables. Long after her speculative ideas were rejected by the male dominated physics community she would be awarded a posthumous

Nobel Prize when Jordan realized that her mathematical speculations provided the key to the mysterious behavior of atomic systems.

The point of this fairy story is that the idea of complementarity is in fact quite natural from the point of view of classical logic, once it is embedded in an appropriate mathematical framework. There is nothing in quantum mechanics which is intrinsically probabilistic until one insists on measuring complementary variables. Much of our unease with quantum mechanics stems from the fact that, as a consequence of our own physical characteristics (the fact that the typical classical action in processes familiar to us before the advent of technology is much larger than Planck's constant), we mistook certain complementary variables for elements of the same abelian subalgebra.

Given a solution $|\psi(t)\rangle$ of the Schrodinger equation for any quantum system, we can construct a complete commuting set of (time dependent) observables, which remain sharp throughout the evolution. These are simply the projector on the time dependent state of the system, and any complete commuting set of orthogonal projectors. Normally, we would reject this statement as a mathematical irrelevancy. Here are some of the reasons why:

- For a system with a time independent Hamiltonian, it is often convenient to introduce fundamental variables which allow us to simply describe the high energy (and thus short time) behavior of the system. In particular, the familiar p's and q's of classical mechanics are appropriate for systems whose high energy behavior is described by a Gaussian fixed point. The sharp observables above are not simply related to the Gaussian variables.

- For systems which are under true experimental control, we like to do repeated experiments with different initial conditions. The sharp observables defined above depend on the initial condition and thus do not provide a convenient way of characterizing all of the experiments we do on the system in a universal framework.

- A related problem is that the the measurements we actually make with external probes on an isolated system have no simple relation to the sharp observables.

The acute reader will have realized that none of these objections apply to the discussion of the universe as a quantum system. The conventional objection that the universe only happens once is precisely the reason that the second objection to sharp observables is irrelevant. Observation shows us that the universe does not have a time independent Hamiltonian, and so the first objection is irrelevant as well. Indeed, our discussion of the Big Bang in the body of the paper suggests that at very early times the state of the system may be characterized in terms of an integrable CFT. At very late times, the

high energy behavior is surely dominated by black hole states, or if we live in an AsDS space, by states near the horizon. Neither of these regimes seems to have a standard classical description. The final objection to the sharp observables for the universe has more to do with our own limitations than with limitations of the applicability of quantum theory to the description of the universe. The conditions for the existence of independent (let alone intelligent) complex systems which can do measurements on pieces of the universe without affecting other parts of it, are very special. In the case of an asymptotically FRW universe such systems can exist in the asymptotic future but are unlikely to exist near the Big Bang. If the universe is AsDS such systems do not even exist in the asymptotic future (eventually everything either collapses into a black hole or is thermalized by the background DeSitter radiation). It is not surprising then that the natural quantities measured by these approximately isolated systems are not compatible with the classical state of the universe as defined by $|\psi(t)\rangle$. Mathematically, we can view the universe as evolving deterministically in a classical state determined as above in terms of projectors on and orthogonal to its wave function. But these classical observables are not measurable by the kinds of apparati that can be constructed out of subsystems to which the approximate notion of locality applies. The probabilistic nature of the universe as we view it is a characteristic of the nature of the things we can measure rather than of the universe itself.

Acknowledgments

We would like to thank, D.Gross, L.Motl, L.Susskind, and particularly R.Bousso for illuminating conversations about cosmology and the holographic principle. The work of T.Banks was supported in part by the Department of Energy. The work of W.Fischler was supported in part by the Robert A. Welch Foundation and NSF Grant PHY-0071512.

References

[1] S. Gubser, *Curvature Singularities: the Good, the Bad and the Naked*, hep-th/0002162.

[2] D. Finkelstein, *Space-Time Code 2*, Physical Review D5, 320, (1972); *Space-Time Code 3*, Physical Review D5, 2922, (1972); *Space-Time Code IV*, (with G. Frye, L. Susskind), Physical Review D9, 2219, (1974); *Space-Time Code V*, Physical Review D9, 2231, (1974), *Space-Time Code 6*, Print 74-1427 (Yeshiva); R. Sorkin, L. Bombelli, J. Lee, D. Meyer, Physical Review Letters 59, 521, (1987); R. Sorkin, D. Rideout, *Evidence for a Continuum Limit in Causal Set Dynamics*, gr-qc/003117; *A Class of Sequential Growth Dynamics for Causal Sets*, Physical Review D61, 024002, (2000), gr-qc/9904062; L. Smolin, T. Jacobson, *Nonperturbative Quantum Geometries*, Nucl. Phys. B299, 295, (1988); L. Smolin, C. Rovelli, A. Ashtekar, *Weaving a Classical Geometry with Quantum Threads*, Physical Review Letters, 69, 237, (1992), hep-th/9203079; L. Smolin, C. Rovelli, *Spin Networks and Quantum Geometry*, Physical Review, D52, 5743, (1995), gr-qc/9505006; L. Smolin, *Linking Topological Quantum Field Theory and Nonperturbative Quantum Gravity*, J. Math. Phys. 36, 6417, (1995), gr-qc/9505028; L. Smolin, F. Markopoulou, *Causal Evolution of Spin Networks*, Nucl. Phys. B508, 409, (1997), gr-qc/9702025; L. Smolin, Y. Ling, *Holographic Formulation of Quantum Supergravity*, hep-th/0009018; L. Smolin, F. Markopoulou, *Holography in a Quantum Space Time*, hep-th/9910146; L. Smolin, *Holographic Formulation of Quantum General Relativity*, Phys. Rev. D61, 084007, (2000); F. Markopoulou, *Insider's Guide to Quantum Causal Histories*, Nucl. Phys. Proc. Suppl. 88, 308, (2000), hep-th/9912137; *Quantum Causal Histories*, Class. Quant. Gravity 17, 2059, (2000), hep-th/9904009.

[3] T. Banks, *QuantuMechanics and CosMology*, Talk given at the festschrift for L. Susskind, Stanford University, May 2000; *Cosmological Breaking of Supersymmetry?*, Talk Given at Strings 2000, Ann Arbor, MI, hep-th/0007146.

[4] S. Gubser, I. Klebanov, A. Polyakov, *Gauge Theory Correlators From Non Critical String Theory* , Phys.Lett.B428:105-114,1998, hep-th/9802109 ;E. Witten, *Anti-De Sitter Space and Holography* , Adv.Theor.Math.Phys.2:253-291,1998, hep-th/9802150 ; T. Banks, M. Green *Nonperturbative Effects in $AdS_5 \times S^5$ String Theory and D=4 SUSY Yang Mills*, JHEP 9805:002, hep-th/9804170 .

[5] G. t' Hooft, *Dimensional Reduction In Quantum Gravity*, gr-qc/9310026; C. Thorn, *Reformulating String Theory With The 1/N Expansion*, hep-th/9405069; L. Susskind, *The World as a Hologram*, J.Math.Phys. 36 (1995) 6377, hep-th/9409089.

[6] C. Thorn, *op. cit.*

[7] T. Banks, W. Fischler, S. Shenker, L. Susskind *M-theory as a Matrix Model: A Conjecture*,Phys.Rev.D55:5112-5128,1997, hep-th/9610043

[8] R. Bousso*A Covariant Entropy Conjecture* , JHEP 9907 (1999) 004, hep-th/9905177 ;*Holography in General Space Times*, JHEP 9906 (1999) 028, hep-th/9906022; *The Holographic Principle for General Backgrounds*,Class.Quant.Grav. 17 (2000) 997, hep-th/9911002

[9] T. Banks *Supersymmetry and Space Time* Talk given at the Strings at the Millenium Conference, CalTech-USC, January 5, 2000.

[10] T. Banks *Supersymmetry and Holography* In Preparation.

[11] R. Bousso, *Positive Vacuum Energy and the N Bound* ,hep-th/0010252.

[12] W. Fischler, L. Susskind *Holography and Cosmology*,hep-th/9806039.

[13] T. Banks, W. Fischler, L. Motl *Duality vs. Singularities*, JHEP 9901 (1999) 019, hep-th/9811194; T. Banks, L. Motl *On the Hyperbolic Structure of Moduli Space With Sixteen SUSYs*, JHEP 9905:015,1999, JHEP 9905:015,1999.

[14] A. Tseytlin, C. Vafa, *Elements of String Cosmology*, Nucl.Phys. B372 (1992) 443, hep-th/9109048; B. Greene *The Elegant Universe*, Norton, 1999,Chapter 10.

[15] V. Lapchinsky,V. Rubakov, *Canonical Quantization of Gravity and Quantum Field Theory in Curved Space-Time*, Acta Phys.Polon.B10:1041-1048,1979 ; T. Banks, W. Fischler, L. Susskind *Quantum Cosmology in 2+1 Dimensions and 3+1 Dimensions*, Nucl.Phys.B262:159,1985; T. Banks *TCP, Quantum Gravity, the Cosmological Constant and All That*,Nucl.Phys.B249:332,1985.

[16] E. Witten, Talk at the Strings 2001 Conference, Tata Institute, Mumbai, India, January 2001.

[17] R. Figari, R. Hoegh-Krohn, C. Nappi, *Interacting Relativistic Boson Fields in the DeSitter Universe With Two Space-Time Dimensions*, Commun.Math.Phys.44:265-278,1975 ; N. Birrel, P. Davies *Quantum Field Theory in Curved Spacetime*, Cambridge University Press, 1982.

[18] L. Susskind, L. Thorlacius, J. Uglum, *The stretched horizon and black hole complementarity*, Phys. Rev. D48, (1993), 3743, hep-th/9306069; G. 't Hooft, C.R. Stevens, B.F. Whiting, *Black hole evaporation without information loss*, Class. Quantum Grav. 11, 621, (1994), gr-qc/9310006; G. 't Hooft, *Quantum information and information loss in general relativity*, in Quantum Coherence and Decoherence, K.Fujikawa, Y.A.Ono, Eds., Elsevier Science B.V., 273 (1996), gr-qc/9509050; *The scattering matrix approach for the quantum black hole: an overview*, J. Mod. Phys. A11, 4623, (1996), gr-qc/9607022..

[19] E. Farhi, A. Guth, *An obstacle to creating a universe in the laboratory*, Phys. Lett. B183, 149, (1987).

[20] G. 't Hooft, *The self-screening Hawking atmosphere. a different approach to quantum black hole microstates*, Nucl. Phys. Proc. Suppl. 68, 174, (1998), gr-qc/9706058.